PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2869. Proposed by the late L. G. WELD.

The successive segments of a broken right line are represented by the successive terms of the harmonic progression, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, *ad infinitum.* Each segment makes with the preceding a given angle θ. What is the distance and what is the direction of the limiting point (if there be such) from the initial point of the first segment?

2870. Proposed by WARREN WEAVER, University of Wisconsin.

A pendulum bob of mass m is attached to one end of a weightless and inextensible string of length l and swings as a conical pendulum with an angular velocity ω_1 about a vertical line through a fixed point to which the other end of the string is attached. If the angular velocity is increased to ω_2, the height through which the bob rises is independent of the length l. Consider, then, a very long and a very short pendulum. Suppose they are each swung first with an angular velocity ω_1 and then with a larger angular velocity, ω_2, the difference between these two values being great enough so that the longer pendulum rises through a height greater than the length of the shorter pendulum. According to the above result the shorter one should rise through this same height, which is obviously impossible. Explain this apparent paradox.

2871. Proposed by the late L. G. WELD.

Weight being disregarded, a package may be admitted to the parcels post if the length plus the greatest girth, measured transversely to the length, does not exceed 72 inches. What is the size of the smallest square window through which all admissible rectangular boxes can be passed?

2872. Proposed by W. D. LAMBERT, U. S. Coast and Geodetic Survey.

The rectangular coördinates of a point P at the time t are given by the equations

$$x = k \cos \gamma \cos (nt - \alpha), \qquad y = k \sin \gamma \cos (nt - \beta),$$

where k, γ, n, α, and β are constants and γ is taken in the first quadrant. An auxiliary angle δ, in the first quadrant, is defined by the equation

$$\sin \delta = \sin 2\gamma \cdot |\sin (\alpha - \beta)|.$$

(1) Show that P describes an ellipse the lengths of whose semi-axes are $k \cos \frac{1}{2}\delta$ and $k \sin \frac{1}{2}\delta$ and the inclinations of whose axes to the x-axis are given by

$$\tan 2\theta = \tan 2\gamma \cos (\alpha - \beta).$$

(2) Show that the time T when P is at the end of an axis is given by

$$\tan (2nT - \alpha - \beta) = \cos 2\gamma \tan (\alpha - \beta).$$

(3) Obtain criteria for distinguishing between the major and minor axes and for the direction of rotation of P and show that the quantities 2γ, $\alpha - \beta$, δ, 2θ and $2nT - \alpha - \beta$ are simply related as the parts of a right spherical triangle, thus providing a partial check on the computation.

2873. Proposed by D. H. RICHERT, Bethel College, Newton, Kan.

At B is the enemy's battery. At M_1 a battery is to be placed to silence B. Listening posts are installed at M_1, M_2, M_3, all provided with stop-watches. From the maps at hand, the three sides of the triangle $M_1M_2M_3$ are known. B is not visible from any one of the points M_1, M_2, M_3.

The sound of a gun fired at B reaches M_1 at the time T, and M_2 at the time $T + \tau_1$ sec., and it reaches M_3 at the time $T + \tau_2$ sec. How far is B from M_1?

2874. Proposed by J. L. RILEY, Stephenville, Texas.

Show that the equation,

$$x^n + ax^{n-1} + bx^{n-2} + \cdots + k = 0,$$

has some imaginary roots if $a^2 - 2b < n\sqrt[n]{k^2}$; $a, b, \cdots k$ are supposed real.

Note. This result is a particular case of a theorem contained in a paper published a few years ago.—EDITORS.

2875.

Show how to draw through a given point a straight line dividing a given triangle into two parts of equal area.

Three cases of this well-known problem (proposed by an Association member) were discussed by Euclid over two thousand years ago. A new solution is sought. —EDITORS.

PROBLEMS—NOTES

1. Mr. R. M. Winger, of the University of Washington, suggested the problem: "Six of the points of the Feuerbach circle of a triangle, namely, the middle points of the sides of the triangle and the middle points of the junctions of the opposite vertices with the orthocenter, lie in pairs at the extremities of diameters." This result is the basis of several proofs that Feuerbach's circle goes through the six points noted above as well as through the feet of the altitudes. See, for example, J. Casey, *Sequel to . . . the Elements of Euclid*, third edition, 1884, p. 58.

H. P. M.

2. The following problem was proposed and solved in the *Journal of the Indian Mathematical Club*, 1910: "All the significant figures except four in a sum of division were replaced by dots, and the result was:

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It is required to recover the lost figures."

Similar problems with "four fours" and "seven sevens" are due to Mr. W. E. H. Berwick, lecturer in University College, Bangor, England:

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